Quick Quiz for AP Review Solutions
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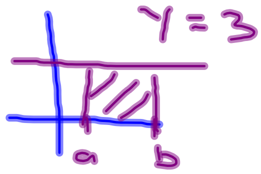
① $\int_a^b f(x)\,dx = a + 2b$

$\int_a^b (f(x) + 3)\,dx$

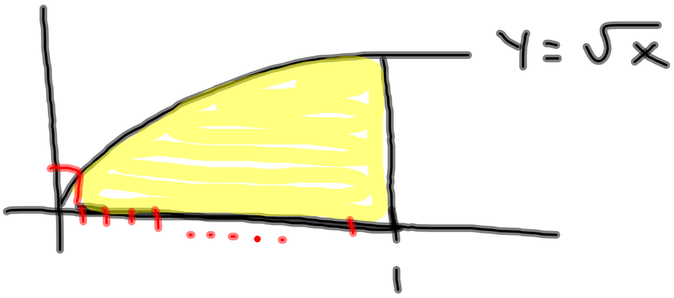
$= \int_a^b f(x)\,dx + \int_a^b 3\,dx$



$= a + 2b + 3(b - a)$

$= 5b - 2a$

$$\frac{1}{20}\left(\sqrt{\frac{1}{20}}+\sqrt{\frac{2}{20}}+\cdots+\sqrt{\frac{20}{20}}\right)$$



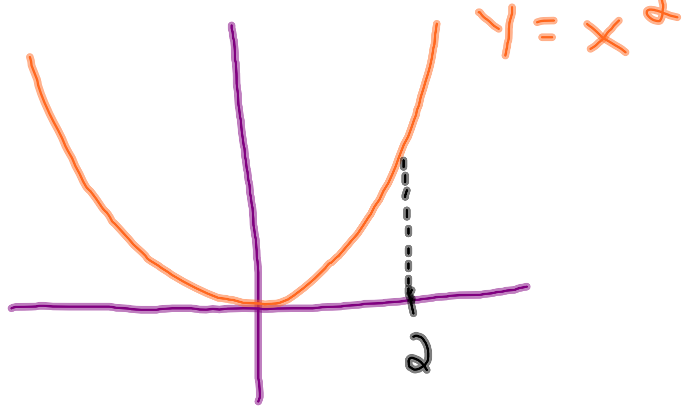
Let $\Delta x = \frac{1}{20}$ ie. $n = 20$

then RRAM is given by ...

$$A = \frac{1}{20}\cdot\sqrt{\frac{1}{20}} + \frac{1}{20}\cdot\sqrt{\frac{2}{20}} + \cdots + \frac{1}{20}\sqrt{\frac{20}{20}}$$

$$= \frac{1}{20}\left(\sqrt{\frac{1}{20}}+\sqrt{\frac{2}{20}}+\cdots+\sqrt{\frac{20}{20}}\right)$$

$$\int_2^2 x^2\,dx = 0$$



$f''(x) = 6x + 12$

a) at $(0,-5)$ the tangent to $f(x)$ is given by $4x - y = 5$, or $y = 4x - 5$.

$\therefore f'(0) = 4$

$f'(x) = 3x^2 + 12x + C$

$f'(0) = 3(0)^2 + 12(0) + C = 4$

$C = 4$

$\therefore f'(x) = 3x^2 + 12x + 4$

$f(x) = x^3 + 6x^2 + 4x + C$

$f(0) = -5$

$\therefore 0^3 + 6(0)^2 + 4(0) + C = -5$

$C = -5$

$\underline{f(x) = x^3 + 6x^2 + 4x - 5}$

b) $av(f) = \frac{1}{1--1} \int_{-1}^{1} (x^3 + 6x^2 + 4x - 5)\,dx$

$= \frac{1}{2}\left[\frac{1}{4}x^4 + 2x^3 + 2x^2 - 5x\right]_{-1}^{1}$

$= \frac{1}{2}\left[\left(\cancel{\tfrac{1}{4}} + 2 + \cancel{2} - 5\right) - \left(\cancel{\tfrac{1}{4}} - 2 \cancel{+2} + 5\right)\right]$

$= \frac{1}{2}(2 + 2 - 5 - 5)$

$= \frac{1}{2}(-6)$

$= -3$

The FUNdamental Theorem of Calculus

FTC - Part I

If f is a continuous function on [a, b], then the function

$$F(x) = \int_a^x f(t)\,dt$$

has a derivative at every point x in [a, b] and

$$F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

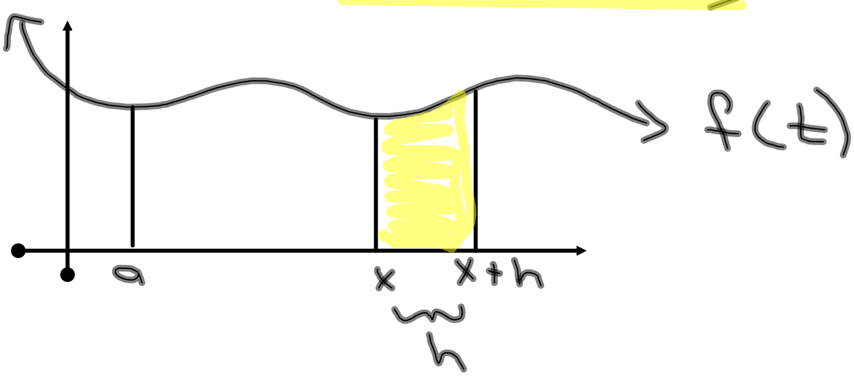
Proof: $\frac{dF}{dx} = \lim_{h \to 0} \frac{F(x+h) - F(x)}{h}$

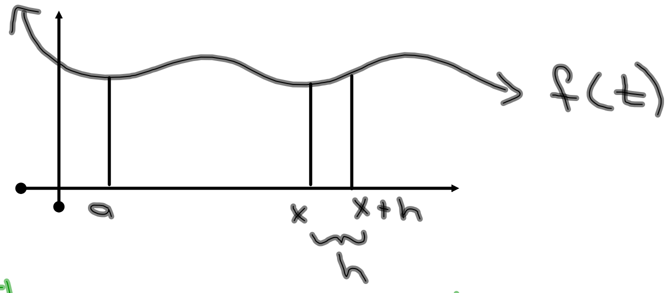
(defn of derivative)

$$= \lim_{h \to 0} \frac{\int_a^{x+h} f(t)\,dt - \int_a^x f(t)\,dt}{h}$$

$$= \lim_{h \to 0} \frac{\int_x^{x+h} f(t)\,dt}{h}$$

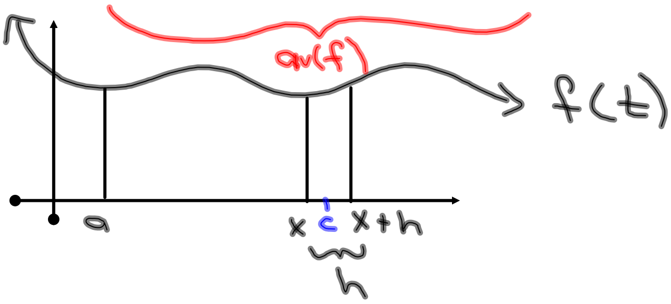
$$= \lim_{h \to 0} \left[\frac{1}{h}\int_x^{x+h} f(t)\,dt\right]$$





The MVT for definite integrals says $\exists$ c in [x, x+h] such that

$$f(c) = \frac{1}{h}\int_x^{x+h} f(t)\,dt$$

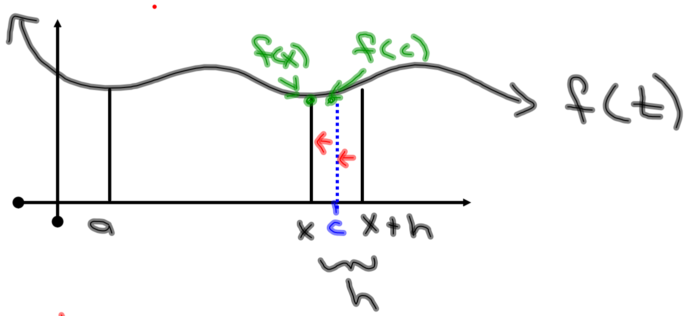


Then...

$$\lim_{h\to 0}\left[\frac{1}{h}\int_x^{x+h} f(t)\,dt\right]$$

$$= \lim_{h\to 0} f(c) \quad \text{for some } c \text{ in } [x, x+h]$$

Now, what happens as $h \to 0$?



As $h \to 0$, $c \to x$

and $f(c) \to f(x)$

So $\lim_{h\to 0} f(c) = f(x)$

$$\boxed{\frac{d}{dx}\int_a^x f(t)\,dt = f(x)}$$

use FTC-I to evaluate..

$$\frac{d}{dx}\int_0^x \cos(t)\,dt$$

$\cos(x)$

$$\frac{d}{dx}\int_\pi^x \cos(t)\,dt =$$

$\cos x$

*The lower limit of integration has no effect on the derivative when it is a constant.

$$\frac{d}{dx}\int_x^0 \cos(t)\,dt$$

$$\frac{d}{dx}\int_5^{x^2} \cos t\,dt$$

$$\frac{d}{dx}\int_5^{\cos x} \cos(t)\,dt$$

$$\frac{d}{dx}\left(\int_0^{x^2} 2t^3-5t+1\,dt\right)$$

$$\frac{d}{dx}\int_{5}^{x}\left(t^2+7t-1\right)dt$$

$$\frac{d}{dx}\int_{x}^{5}\left(t^2-5t\right)dt$$

$$\frac{d}{dx}\int_{4}^{\cos x}\sin t\,dt$$

$$\frac{d}{dx}\left[\int_{4}^{x}(2t^2-7t)\,dt\right]$$

$$\frac{d}{dx}\left[\int_{2x}^{4}(2t^2-7t)\,dt\right]$$

$$\frac{d}{dx}\left[\int_{7}^{x^2}\cos t\,dt\right]$$

$$\frac{d}{dx}\left[\int_{x^2}^{x^3} \cos(t)\,dt\right]$$

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